Time Based Correspondences Using Sonar Scan Matching

Martí Sales, Yolanda González and Antoni Burguera
Dept. Matemàtiques i Informàtica - Universitat de les Illes Balears
Ctra. Valldemossa Km. 7,5 - 07122 Palma de Mallorca (SPAIN)
noesnaidel@gmail.com, {antoni.burguera,y.gonzalez}@uib.es

Abstract

Scan Matching (SM) *is a technique to estimate the robot pose by computing the overlap between two successive range scans.*

Several SM *algorithms have been developed in last decades and, although they work well enough with laser data, they are less powerful when treating data from ultrasonic sensors, which are more scattered and noisy.*

In this paper we present a new SM *algorithm whose main contribution is a time measurement instead of distance between the scan data points, particularly focused on ultrasonic applications.*

1. Introduction

For a mobile robot to execute long-term missions, it has to provide some localization capabilities. Most of methods are based on matching recent sensory information against prior knowledge of the environment and introduce geometric constraints, such as lines and corners, typical in indoor scenarios [1] [4] [5]. Although these techniques may reduce the computational complexity, they present some problems in non-structured environments. To confront this problem, some authors determine the robot displacement by matching up successive sets of range measurements, called scans. This technique is known as *Scan Matching* [3] [8].

The most popular algorithm to perform *scan matching* is the ICP (*Iterative Closest Point*) [6], which uses Euclidian distance to establish the correspondences between the two sets of range readings. Although this algorithm provides good estimates, ICP is weak for a relatively large rotation error. Among others, IDC (*Iterative Dual Correspondence*) deals with the rotation problem by establishing two sets of correspondences, one dealing with the translation and the other with the rotation, MbICP (*Metric-based ICP* **??** defines a new measure that accounts for traslationa and rotation, or spIC (*sonar probabilistic Iterative Correspondence*) [2] which takes into account the exteroceptive sensor imprecisions by computing statistical compatibility between the scan points.

The algorithm proposed here is a novel approach that uses time to measure distances and to establish correspondences. In other words, the minimum distance between two points is not the shortest one, but that one which takes less time to be covered. That is why the algorithm is called *Iterative Earliest Points* (*IEP*).

2. *Scan Matching*: definitions and notation

Scan Matching consists in an iterative process to estimate the robot displacement and rotation that maximize the overlap between two consecutive sensor scans. However, when performing *sonar SM* a previous step –far from the scope of this study– of grouping the readings from successive robot positions is necessary in order to avoid having range scans with a few number of readings [2].

Let $S_{ref} = \{q_1, q_2, ..., q_n\}$ be a set of n points gathered at frame A, which is called the *reference scan*.

Let $S_{cur} = \{p_1, p_2, ..., p_m\}$ be a set of m points gathered at frame B, which is called the *current scan*.

Let $x^A_{B_k}$ be the scan matching estimate of the relative position between the frames A and B at the iteration k, being $k = 0$ at the beginning of the process.

In a general *SM* algorithm, at iteration $k > 0$, the following three steps are executed:

- S_{cur} is expressed respect to frame A using $x^A_{B_{k-1}}$. Let $S'_{cur} = \{p'_1, p'_2, ..., p'_m\}$ be the set of trasformed points. That is $p'_i = x^A_{B_{k-1}} \oplus p_i, \forall p_i \in S_{cur}$.
- For each point $q_i \in S_{ref}$, the corresponding p'_j is determined as the closest point in S'_{cur} whose distance $d\left(q_i, p'_j\right)$ is below a certain threshold d_{min}. The set of correspondences $C_k = \ ((i,j) \,\|1 \leq i \leq n, 1 \leq j \leq m)$ is obtained.
- The motion estimate $x^A_{B_k}$ that minimizes the sum of squared distances between q_i and $x^A_{B_k} \oplus p_j$ is computed, being $(i,j) \in C_k$. The error present at iteration k, e_k, is defined as the mentioned sum of squared distances.

When convergence is achieved, the algorithm ends with the solution $x^A_{B_k}$. Otherwise, the process iterates. This study, consideres that convergence is achieved when the error change ratio between two consecutive iterations is below a threshold: $\frac{|e_k - e_{k-1}|}{e_{k-1}} \leq \delta$.

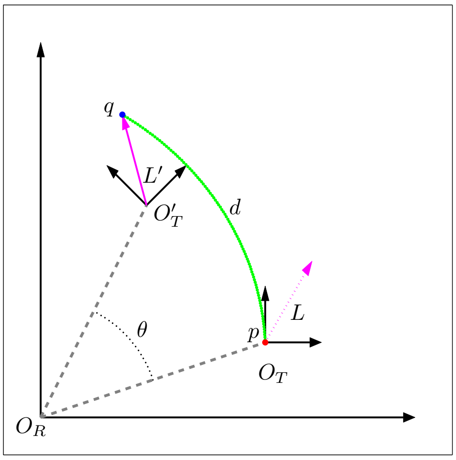

Figure 1. Trajectory model description

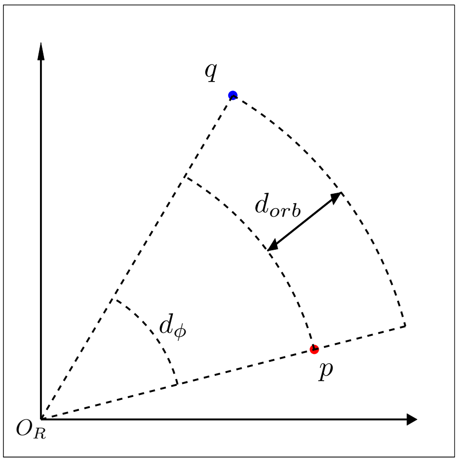

Figure 2. d_{orb} and d_ϕ definitions

3. The Robot Model

Since this study deals with distance and time, we should adopt the concept of *speed*. Then it is necessary to build the model to describe the robot's motion.

The *IEP* algorithm describes any trajectory between two given points p and q) as a composition of two independent and simultaneous movements: a rotation at angular speed ω and a straight traslation at speed v. This composition of two movements is illustrated in Figure 1 where the rotation, traslation and resulting trajectory are shown. There are two coordinate systems O_R and O_T: the point p is traslated with respect O_T and O_T rotates with respect O_R. So, in this example, we can see that the composition of the rotation angle θ with the traslation L results on the trajectory d that allows the point p reach the point q.

Given two points p and q in a plane, in cartesian $(x_p, y_p), (x, y)$ and polar $(r_p, \alpha_p), (r_q, \alpha_q)$ coordinates respectively, the *orbital distance* and *angular distance*, illustrated in Figure 2, are defined as follows:

$$d_{orb} = r_q - r_p \qquad d_\phi = \alpha_q - \alpha_p$$

being d_ϕ normalized between $-\pi$ and π. By using this notation, two nominal *times* are defined as:

$$t_{t0}\left(p, q\right) = \frac{|d_{orb}|}{v} \qquad t_{r0}\left(p, q\right) = \frac{|d_\phi|}{\omega}$$

4. The *IEP* Algorithm

The goal of the algorithm is to find the optimal trajectory $M = [T_x, T_y, \phi]$ that ensures the minimum time $t_{min}\left(p, q\right)$, according to the definition of the robot's model. T_x and T_y are the traslation components and ϕ is the rotation angle to be applied, which will always satisfy $|\phi| \leq |d_\phi|$.

By knowing ϕ it is possible to obtain T_x and T_y as it is shown below:

$$T_x = q_x - p'_x \qquad T_y = q_y - p'_y$$

where

$$\begin{bmatrix} p'_x \\ p'_y \end{bmatrix} = \begin{bmatrix} \cos\left(\phi\right) & -\sin\left(\phi\right) \\ \sin\left(\phi\right) & \cos\left(\phi\right) \end{bmatrix} \cdot \begin{bmatrix} p_x \\ p_y \end{bmatrix}$$

If $t_{t0} \geq t_{r0}$, then t_{min} and ϕ are calculated as

$$t_{min}\left(p, q\right) = t_{t0}\left(p, q\right) \qquad \phi = d_\phi$$

Otherwise, it means that it is possible to use the extra rotation time in an extra traslation. Thanks to this, the minimum time needed can be reduced. Then, the time needed by both movements (t_r and t_t) must be the same:

$$t_r(p, p') = t_t(p', q) \tag{1}$$

Moreover, we have the following equations:

$$t_r(p, p') = \frac{|\phi|}{\omega} \tag{2}$$

$$t_t(p', q) = \frac{\sqrt{\left(q_x - p'_x\right)^2 + \left(q_y - p'_y\right)^2}}{v} \tag{3}$$

By replacing (2) and (3) in (1)

$$\frac{|\phi|}{\omega} = \frac{\sqrt{A^2 + B^2}}{v} \tag{4}$$

is obtained, where

$$A = q_x - \cos\left(\phi\right) p_x + \sin\left(\phi\right) p_y$$

$$B = q_y - \sin\left(\phi\right) p_x - \cos\left(\phi\right) p_y$$

Since (4) has the form $x = f(x)$ it is necessary to apply a numerical method to find the value of ϕ that satisfies (4). In terms of convergence and speed the *Secant Method* has bee chosen.

The *IEP* equidistant curves, which represent the collection of points equally time-spaced from an original given point, are depicted in Figure 3(a).

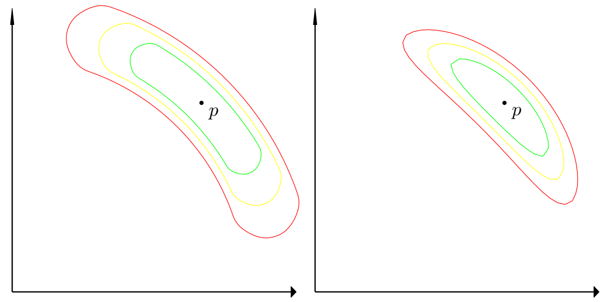

Figure 3. IEP (left) and IEP2 (right) equidistant curves

4.1. Simplification of time calculation

In this paper we also propose a simplification of the minimum time calculation that allows a faster computation. It is estimated as follows:

$$t_{min}(p,q) = \sqrt{t_{t0}^2 + t_{r0}^2}$$

Henceforth, the algorithm will be called *IEP2* when using the faster calculation. The *IEP2* equidistant curves are represented in Figure 3(b). As it can be observed, the *IEP2* curve presents a very similar shape to the real reading dispersion of an ultrasonic range sensor [7].

4.2. Pose estimation

After calculating the minimum time between each pair of points in the data scans, the algorithm sets correspondences between *earliest* points, *i. e.* between the points (p_u, q_v) with minimum $t_{min}(p_u, q_v)$, and builds the matrix (C) as follows:

$$C = \begin{bmatrix} u_1 & \cdots & u_n \\ v_1 & \cdots & v_n \\ M_1 & \cdots & M_n \end{bmatrix} \quad (5)$$

Where, $\forall i \in \{1, n\}$, n is the number of correspondences, u_i is an index to a point in S_{ref}, v_i is an index to a point in the S_{cur} and M_i is the trajectory $[T_{xi}, T_{yi}, \phi_i]$ calculated for the i^{th} correspondence. The example in Figure 4 shows the set of correspondences for *ICP* (top) and for *IEP* (bottom). It is easy to appreciate that *IEP* distance is rather more sensitive to rotation than the euclidean one. Once (5) is built, the pose estimation that we propose is obtained as stated below:

$$x_{B,k+1}^A = \begin{bmatrix} \delta_x \\ \delta_y \\ \phi \end{bmatrix} \quad (6)$$

being

$$\delta_x = \frac{\sum_{i=1}^n T_{xi}}{n};\ \delta_y = \frac{\sum_{i=1}^n T_{yi}}{n};\ \phi = \frac{\sum_{i=1}^n \phi_i}{n}$$

Although (6) is not obtained from a minimization process, the implemented experiments reveal that the sum of squared times is reasonably reduced after each iteration.

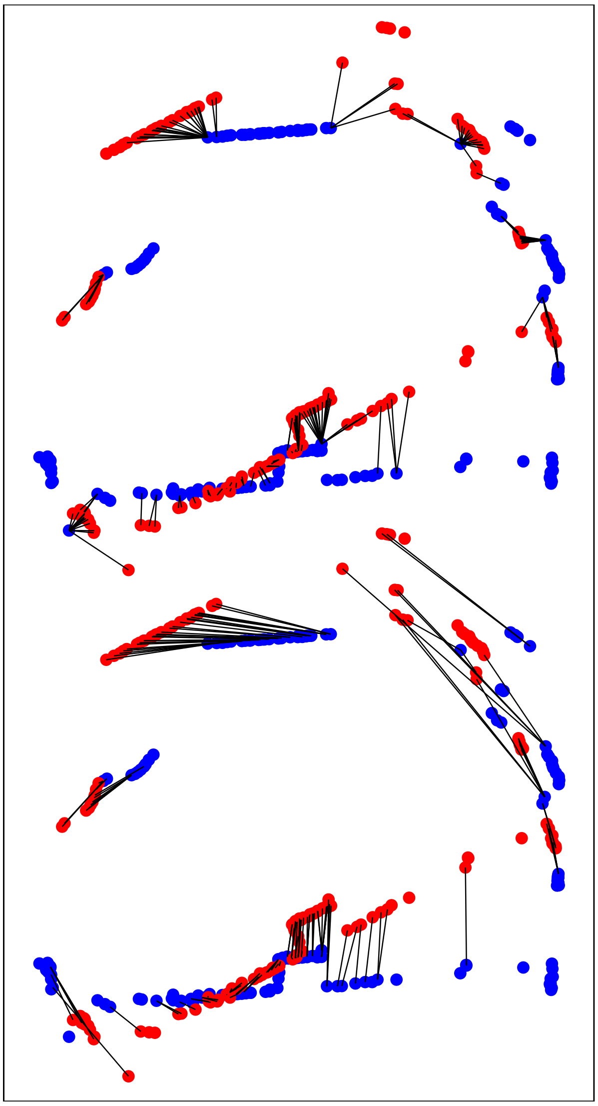

Figure 4. ICP vs. IEP correspondences

4.3. Convergence acceleration

Experiments have shown that the final estimated solution $x_{B_k}^A$ consists on a monotonous addition of previous estimations. Since all these previous estimations have the same rotation and traslation directions, this paper also provides a method to accelerate the convergence that consists in multiplying every estimation by a proportional factor which tends to be zero when the iteration process ends. Thanks to this, it is possible to reach the same final solution with a less number of iterations, which is translated in a faster computation time.

So, the new estimation $x'^A_{B_{k+1}}$ is obtained as follows:

$$x'^A_{B,k+1} = x^A_{B,k+1} \cdot \left(1 + \frac{|e_k - e_{k-1}|}{e_{k-1}}\right)$$

The Figure 5, where the sum of time squared with and without acceleration (red and blue lines respectively) are represented, shows the behaviour of this method.

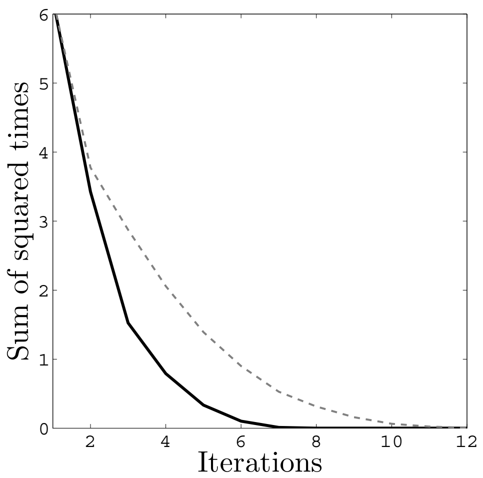

Figure 5. Convergence comparison

5. Experimental results

All the algorithms mentioned in this paper have been tested and compared through more than 150.000 experiments with 3 different scan sizes and 2 error types (absolut and oddometric) at 5 gradually sparsed levels. The introduced traslation error varies from 5 to 25 cm while the introduced rotation error varies from 5 to 45 degrees (both in absolute values). In order to achieve a reliable comparison we define the next main indicators to be observed: *it* is the number of iterations needed to converge to a solution (tolerance $x_B^A < (.075m, .075m, .075rad)$) and *hit* is the percentage of right solutions.

A first comparative of *hits* vs. scan sizes is shown in Table 1. It is observed that, in all cases, *IEP* and *IEP2* algorithms return the best results in front the rest of algorithms. The larger the scan size the larger the number of correspondences, so the algorithm becomes more stable and the *hits* ratio is increased.

Algorithm	Small	Medium	Large
IEP2	$41,5\%$	$48,5\%$	49%
IEP	45%	48%	48%
spIC	40%	44%	35%
IDC	26%	34%	46%
MbICP	23%	24%	31%
ICP	27%	$23,5\%$	24%

Table 1. hit vs. scan sizes

A second analysis, depicted in Table 2, shows the global *hit* ratio of each algorithm for the whole set of experiments. Now again, *IEP* and *IEP2* algorithms lead the comparison with the higest *hit* ratio, being the only two above the 45%. Finally, regarding the algorithms convergence, a new indicator *hits per iteration* (*hpi*) is defined as $hpi = hit/it$ and it is better as higer is its value. The results are shown also in Table 2. We conclude that the two algorithms presented in this document have the best ratio of *hits* against the number of iterations needed to converge.

Algorithm	*hit*	*hpi*
IEP	47%	$5,63$
IEP2	46%	$4,92$
spIC	41%	$4,45$
IDC	34%	$3,68$
ICP	25%	$1,62$
MbICP	25%	$2,98$

Table 2. Global *hit* and *hpi* ratios

6. Conclusion and future work

In this paper, the *IEP* algorithm is presented as a new solution for *SM* focused on ultrasonic range sensors, which deals with the low angular resolution and sparsy readings from such kind of devices. It is also presented a simplification of the algorithm, called *IEP2*, which provides an improvement in terms of computaton speed and obtaining similar results. Finally, a method to accelerate the convergence is exposed, being demonstrated that the number of iterations to reach a solution has been reduced.

Some aspects of the algorithm are still under study, primarily trying to find a mathematical minimization process to prove that every estimation obtained is the optimal one. Moreover, a calibration process for v and ω is being studied in order to achieve better results. One possible way could be using the real robot speed parameters on each instant when the *SM* is executed.

Acknowledgements

This research is partially supported by DPI 2008-06548-C03-02 and FEDER funding.

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