A nonlinear fault-tolerant thruster allocation architecture for underwater remotely operated vehicles

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Abstract: This paper proposes an actuator failure tolerant robust control scheme for underwater Remotely Operated Vehicles (ROVs). A reduced order observer has been introduced for estimating the ROV velocities and a sliding mode control law has been developed using the available position measurements and the velocity estimates provided by the observer to achieve output regulation. A thruster failure is shown to be detectable simply checking the presence of any deviation of the observed sliding surfaces. Moreover, an isolation policy for the failed thruster is proposed. Finally, control reconfiguration is performed exploiting the inherent redundancy of actuators with the implementation of an adaptive input allocation architecture. An extensive simulation study has been performed, supporting the effectiveness of the proposed approach.

1. INTRODUCTION

Unmanned Underwater Vehicles (UUVs) are a cost-effective solution for performing complex tasks in the underwater environment without risking human life, such as environmental data gathering, transportation of assembling modules for submarine installations, inspection of underwater structures. With increasing mission durations in complex marine applications, one of the primary concerns is the failure occurrence on the actuators (Fossen, 1994; Sarkar et al., 2002). When actuator failures occur and result in abnormal operations, the only present solution is to abort the mission, and use a damage control to make UUVs surface (Isermann and Ballé, 1997). Therefore, the problem of reliability and security of UUVs, especially their ability of actuator fault tolerance, has become a major concern. It is desirable to incorporate a function of actuator fault detection and isolation into the control system, so that it is possible to detect and identify actuator fault and/or failures and estimate their severity, in order to design optimal compensation laws.

This paper presents a fault-tolerant control allocation scheme in the framework of underwater robotics, specifically addressing an underwater Remotely Operated Vehicle (ROV) (Longhi and Rossolini, 1989) used by SNP-progetti (Fano, Italy) in the exploitation of combustible gas deposits at great water depths. The vehicle is equipped with four thrusters, controlling its position and orientation in planes parallel to the sea surface, and is connected with the surface vessel by a supporting cable which controls the vehicle depth and provides power and communication facilities. The control system is composed of two independent parts: the first part, placed on the surface vessel, monitors the vehicle depth and it typically involves a human operator, while the second part is automated and controls the position and orientation of the vehicle in the dive plane. In this paper the attention has been focused on this second part of the control system: the ROV is supposed to move on a plane, with three degrees of freedom. Due to this assumption, the thrusters configuration is redundant. A typical efficient way to exploit such redundancy is the implementation of a control allocation architecture (Johansen and Fossen, 2013), this being a paradigm capable of easily handling constraints and optimization requirements.

The actuator failure diagnosis scheme presented in the paper is composed by the usual modules performing detection and isolation of faults (Gertler, 2002). A reduced order observer has been specifically designed for the ROV exploiting the features of the underwater vehicle, permitting to estimate the underwater vehicle velocities, whose direct measurements are typically difficult to be gathered and poorly reliable. These velocity estimations and the available position measurements, have been then used for developing a robust sliding mode control law (Utkin, 1992), which is able to solve the regulation problem for the ROV positions, with respect to the reference ones. The developed sliding surfaces have been used both for designing a robust ROV control algorithm ensuring plant regulation, and for detecting the thruster failures. Failure detection is here performed simply checking the presence of any deviation of the observed sliding surfaces, which can be due only to the occurrence of a thruster failure. Exploiting the ROV structure, faulty thruster can be successfully isolated. Once the failed actuator has been identified, control reconfiguration is performed using the redundant healthy actuators. While in previous works, such as (Corradini and Orlando, 2014), a supervisor was supposed to be in charge of applying and monitoring the control reconfiguration, the main contribution of the present paper is to present an automated reconfiguration method based on adaptive control allocation. The proposed scheme takes inspiration from well established results on fault-tolerant control allocation (Cristofaro and Johansen, 2014b; Cristofaro et al., 2015; Tohidi et al., 2016) but incorporates also some special features of the particular ROV model.

2. MATHEMATICAL MODEL OF THE ROV

2.1 ROV nonlinear model

The equations describing the ROV dynamics have been obtained from classical mechanics (Longhi and Rossolini,
The ROV considered as a rigid body can be fully described with six degrees of freedom, corresponding to the position and orientation with respect to a given coordinate system. Let us consider the inertial frame \( \mathbf{R}(0, x, y, z) \) and the body reference frame \( \mathbf{R}_a(x_a, y_a, z_a) \) (Conter et al., 1989) shown in Fig. 1. The ROV position with respect to \( \mathbf{R} \) is expressed by the origin of the system while its orientation by the roll, pitch, and yaw angles \( \psi, \theta, \phi \), respectively. Being the depth controlled by the surface vessel, the ROV is considered to operate on surfaces parallel to the \( x - y \) plane. Accordingly the controllable variables are \( x, y \), and the yaw angle \( \phi \). It should be noticed that the roll and pitch angles \( \psi \) and \( \theta \) will not be considered in the dynamic model: their amplitude, in fact, has been proved to be negligible in a wide range of load conditions, and with different intensities and directions of the underwater current as well (Longhi and Rossolini, 1989), (Conter et al., 1989). Therefore, the ROV model is described by the following system of differential equations (Longhi and Rossolini, 1989), (Conter et al., 1989), (Corradini and Orlando, 1997), (Corradini et al., 2011):

\[
\begin{align*}
\dot{\mathbf{x}} &= \mathbf{f}(\mathbf{z}(t), \mathbf{u}(t)) + \Delta \mathbf{f}(\mathbf{z}(t), \mathbf{u}(t)) + \mathbf{g} \mathbf{u}(t), \\
\mathbf{z}(t) &= [z_1, z_2, z_3]^T = [x, y, z]^T, \\
\mathbf{u}(t) &= [u_1, u_2, u_3]^T
\end{align*}
\]

(1)

where \( \mathbf{V}_c = [V_{cx}, V_{cy}]^T \) is the submarine current velocity, \( \mathbf{V} = [V_x, V_y]^T = [(\dot{x} - V_{cx}) (\dot{y} - V_{cy})]^T \), and the expressions of coefficients \( p_i (i = 1, \ldots, 9) \) are reported in (Corradini and Orlando, 1997), (Corradini et al., 2011), where \( M \) is the vehicle mass, \( n \) is the addition mass, \( I_z \) is the vehicle inertia moment around the \( z \) axis, \( i_z \) is the addtional inertia moment, \( M_a \) is the resistance moment of the cable, \( L \) is the cable length, \( T_i \) the vehicle weight in the water, \( W \) the weight for length unit of the cable, \( \rho_w \) the water density, \( C_{de} \) is the drag coefficient of the cable, and \( D_e \) is the cable diameter, \( C_{di} \) is the drag coefficient of the \( i \)-th side wall \( (i = 1, 2) \), \( C_{ri} \) the packing coefficient (depending on the geometrical characteristics of the \( i \)-th side wall \( (i = 1, 2) \), \( C_d \) is the drag coefficient of rotation, \( S \) the packing coefficient of rotation, \( S \) is the equivalent area of rotation, \( r \) is the equivalent arm of action, \( d_i \) \((i = 1, 2, 3) \) are the vehicle dimensions along the \( x_a, y_a \) and \( z_a \) axes, respectively, and \( \phi \) is the angle between the \( x \) axis and the velocity direction of the current. This model is in agreement with models usually proposed in literature for underwater ROV’s moving in the dive plane (Fossen, 1994). The quantities \( T_x, T_y \) and \( M_z \) appearing in (1) are the decomposition of the thrust and the torque provided by the four vehicle thrusters along the axes of \( \mathbf{R} \):

\[
\begin{align*}
T_x &= \cos(\phi) (\tau_1 + \tau_2 + \tau_3 + \tau_4) \cos(\alpha) - \\
& \sin(\phi) (-\tau_1 - \tau_2 + \tau_3 + \tau_4) \sin(\alpha) \\
T_y &= \sin(\phi) (\tau_1 + \tau_2 + \tau_3 + \tau_4) \cos(\alpha) + \\
& \cos(\phi) (-\tau_1 - \tau_2 + \tau_3 + \tau_4) \sin(\alpha) \\
M_z &= (-\tau_1 + \tau_2 - \tau_3 + \tau_4) d_a
\end{align*}
\]

where \( \alpha = \pi / 4 \), \( d_a = (d_x \sin(\alpha) + d_y \cos(\alpha)) \) (see Fig. 1).

2.2 State Space ROV model

Define the vectors \( \mathbf{z}_p = [z_1, z_2, z_3]^T = [x, \dot{x}, \dot{y}]^T \), \( \mathbf{z}_v = [z_4, z_5, z_6]^T = [\dot{z}, \dot{\phi}, \dot{\phi}_c]^T \), the state vector:

\[
\mathbf{z} = [\mathbf{z}_p \; \mathbf{z}_v]^T = [z_1, z_2, z_3, z_4, z_5, z_6]^T
\]

and introduce the input vector \( \mathbf{u} = [u_1, u_2, u_3]^T = [T_x, T_y, M_z]^T \). Moreover, since model parameters and submarine current are not exactly known, bounded uncertainties are taken into account as follows: \( p_i = \hat{p}_i + \Delta p_i \), \( \lvert \Delta p_i \rvert \leq \rho_{p_i}, i = 1, \ldots, 9 \), \( V_{cx} = \hat{V}_{cx} + \Delta V_{cx} \), \( \lvert \Delta V_{cx} \rvert \leq \rho_{V_{cx}} \), \( V_{cy} = \hat{V}_{cy} + \Delta V_{cy} \), \( \lvert \Delta V_{cy} \rvert \leq \rho_{V_{cy}} \), \( \hat{V}_{cx}, \hat{V}_{cy}, \phi \), \( \rho_{V_{cx}}, \rho_{V_{cy}}, \rho_{\phi} \) the nominal values of the parameter and of the submarine current components, respectively, and \( \lvert \Delta \phi, \lvert \Delta V_{cx}, \lvert \Delta V_{cy} \) the corresponding uncertainties, bounded by \( \rho_{\phi}, \rho_{V_{cx}}, \rho_{V_{cy}} \), respectively. Considering the above definitions and equations (1), the following state space model is obtained:

\[
\dot{\mathbf{z}}(t) = \mathbf{f}(\mathbf{z}(t)) + \Delta \mathbf{f}(\mathbf{z}(t), \mathbf{u}(t)) + \mathbf{g} \mathbf{u}(t)
\]

(3)

where

\[
\mathbf{f}(\mathbf{z}) = \begin{bmatrix}
\frac{z_4}{\hat{p}_1} \\
\frac{z_5}{\hat{p}_1} \\
\frac{z_6}{\hat{p}_1} \\
\frac{-\tau_1}{\hat{p}_0} z_0 + \varphi_z(s_3) \\
\varphi_z(z_3)
\end{bmatrix}
\]

(4)

being

\[
\begin{align*}
f_4(z_3) &= \frac{1}{\hat{p}_1} (\hat{p}_2 |z_3| + \hat{p}_3 |x_3|) \\
f_5(z_3) &= \frac{1}{\hat{p}_1} (\hat{p}_2 |s_3| + \hat{p}_3 |c_3|) \\
\varphi_z(z_3) &= \frac{1}{\hat{p}_1} (\hat{p}_4 z_1 + \hat{p}_5 V_{cx} |V_{ci}|) \\
\varphi_5(z_2) &= \frac{1}{\hat{p}_0} (\hat{p}_4 z_2 + \hat{p}_5 V_{cy} |V_{ci}|) \\
\varphi_6(z_3) &= \frac{1}{\hat{p}_0} \left( \hat{p}_8 |V_{ci}|^2 \sin \left( \frac{z_3 - \phi_c}{2} \right) + \hat{p}_9 \right)
\end{align*}
\]

Fig. 1. ROV operational configuration.
with \( \bar{z}_4 = z_4 - \hat{V}_c x, \bar{z}_5 = z_5 - \hat{V}_c y, N_z = \sqrt{\bar{z}_4^2 + \bar{z}_5^2}, c_3 = \cos(z_3), s_3 = \sin(z_3), \hat{V}_c = \sqrt{\hat{V}_{cx}^2 + \hat{V}_{cy}^2}, \phi_c = \arctan\left(\frac{\hat{V}_c y}{\hat{V}_c x}\right), \) and the term \( \Delta f(z,u) = [0,0,0,\Delta \xi_1(z_1, z_3, z_4, T_z), \Delta \xi_2(z_2, z_3, z_5, T_y), \Delta \xi_6(z_3, z_6, M_z)]^T \) can be easily computed considering the uncertainties \( |\Delta p_i| \) \( i = 1 \ldots 9 \), \( \Delta V_{cx}, \Delta V_{cy} \).

3. LOW-LEVEL ROBUST CONTROL DESIGN

3.1 Robust Design of the Reduced Order Observer

In this section, a reduced order observer, specifically designed for the ROV, will be proposed. In fact, while position measurements are usually available and are in general sufficiently reliable, velocity measurements are either difficult to be gathered and poorly reliable.

\[
\xi = [\xi_1 \; \xi_2 \; \xi_3]^T,
\]

Define \( \xi = [\xi_1 \; \xi_2 \; \xi_3]^T \), and consider the following reduced state observer:

\[
\bar{\xi}_1 = -f_4(z_3)N_z \xi_1 + \varphi_4(z_1) + v_1 + v_1
\]

\[
\bar{\xi}_2 = -f_5(z_3)N_z \xi_2 + \varphi_5(z_2) + v_2 + v_2
\]

\[
\bar{\xi}_3 = -\alpha_6 M_z \xi_3 + \varphi_3(z_3) + v_3
\]

with \( \bar{\xi}_1 = \xi_1 - V_c x, \bar{\xi}_2 = \xi_2 - V_c y, \alpha_6 = pr/\rho_0, M = \text{sup}\{z_{max}^2, M_z\}, N_z = \sqrt{(\xi_1 - V_c x)^2 + (\xi_2 - V_c y)^2} \) and \( v_1, v_2, v_3 \) are auxiliary parameters to be specified.

A robust control law, coupled with the above observer, will be here presented aimed at solving the regulation problem for the variables \( z_1, z_2, z_3 \) with respect to reference variable \( z_d = [z_{1d} \; z_{2d} \; z_{3d}]^T \). Define the following sliding surface:

\[
\bar{s} = [\bar{s}_1 \; \bar{s}_2 \; \bar{s}_3]^T = [\xi - z_d] + \Lambda(z_p - z_d) = 0.
\]

with \( \Lambda = \text{diag}([\lambda_i], \lambda_i > 0, i = 1, \ldots, 3) \), and being \( \bar{s} = z_p - z_d \) the tracking error.

Lemma 1. Consider the uncertain ROV model (3). The control law \( u = u_{eq} + u_n \), with:

\[
u_{eq} = g_0^{-1} \begin{bmatrix} f_4(z_3) \bar{\xi}_1 N_z - \varphi_4(z_1) - \lambda_1(z_1 - \bar{z}_{1d}) + \bar{z}_{1d} - v_1 \\ f_5(z_3) \bar{\xi}_2 N_z - \varphi_5(z_2) - \lambda_2(z_2 - \bar{z}_{2d}) + \bar{z}_{2d} - v_2 \\ \alpha_6 M_z \bar{\xi}_3 - \varphi_3(z_3) - \lambda_3(z_3 - \bar{z}_{3d}) + \bar{z}_{3d} - v_3 \end{bmatrix}
\]

\[
u_n = -g_0^{-1} \begin{bmatrix} (\rho_2 + \omega) \text{sign}(\bar{s}_1) \\ (\rho_3 + \omega) \text{sign}(\bar{s}_2) \\ (\rho_4 + \omega) \text{sign}(\bar{s}_3) \end{bmatrix}, \quad \omega > 0
\]

guarantees the asymptotical achievement of a sliding motion on (6) with a finite reaching phase.

Proof. The achievement of a sliding motion on (6) is guaranteed by the following condition:

\[
\bar{s}^T \bar{s} = \bar{s}_1 \left( \xi_1 - \bar{z}_{1d} + \lambda_1(z_1 - \bar{z}_{1d}) \right) + \bar{s}_2 \left( \xi_2 - \bar{z}_{2d} + \lambda_2(z_2 - \bar{z}_{2d}) \right) + \bar{s}_3 \left( \xi_3 - \bar{z}_{3d} + \lambda_3(z_3 - \bar{z}_{3d}) \right) < 0
\]

which can be fulfilled imposing separately three inequalities, i.e. \( \xi_i - \bar{z}_{id} + \lambda_i(z_{i+1} - \bar{z}_{id}) < 0 \), \( i = 1, 2, 3 \). The first inequality gives, e.g.:

\[
\bar{s}_1 \left( -f_4(z_3)N_z \xi_1 + \varphi_4(z_1) + v_1 - \bar{z}_{1d} + \lambda_1(z_4 - \bar{z}_{1d}) \right) < 0
\]

and one gets immediately the controller (7). In particular, selecting \( \omega \) arbitrarily small but positive guarantees a finite reaching phase. Define the observation error as \( e = [e_1 \; e_2 \; e_3]^T = z_v - \xi \). The following result can be given omitting the proof for brevity:

Corollary 2. Consider the uncertain plant model (3) driven by the control law (7). The reduced order observer (5) ensures the robust asymptotical vanishing both of the observation error and of the tracking error designing \( \nu \) as follows:

\[
\nu = -\begin{bmatrix} f_4(z_3) M(z_{max}^4 + |\xi_1|) + \rho_4 \\ f_5(z_3) M(z_{max}^5 + |\xi_2|) + \rho_5 \\ \alpha_6(z_{max}^6 + M|\xi_3|) + \rho_6 \end{bmatrix}
\]

\[
\begin{bmatrix}
\text{sign}(\xi_1 - \bar{z}_{1d} - z_{1d}) \\
\text{sign}(\xi_2 - \bar{z}_{2d} - z_{2d}) \\
\text{sign}(\xi_3 - \bar{z}_{3d} - z_{3d})
\end{bmatrix}
\]

with \( \theta > 1 \).

4. FAULT-TOLERANT CONTROL ALLOCATION

In the scenario considered in this paper, each thruster is an actuator potentially affected by faults. The basic idea is that, whenever a failure is detected and identified, a supervisor performs a control reconfiguration exploiting thrusters redundancy (three thrusters are enough to control the ROV trajectory). In this framework, it is convenient to rewrite the model (3) as follows:

\[
\dot{z} = f(z) + \Delta f(z,u) + \left[ \begin{array}{c}
0_{3x3} \\
\text{diag}(1,1,1)
\end{array} \right] \bar{g}(z_p) \Gamma
\]

with

\[
G = \begin{bmatrix}
1 & 1 & 1 \\
-1 & -1 & 1 \\
1 & 1 & -1
\end{bmatrix}, \quad \tau = \begin{bmatrix}
\tau_1 \\
\tau_2 \\
\tau_3 \\
\tau_4
\end{bmatrix},
\]

\[
\bar{g}(z_p) = \begin{bmatrix}
g_1(z_3) \\
g_2(z_3) \\
g_3(z_3)
\end{bmatrix}
\]

\[
\begin{bmatrix}
c_3 \cos(\alpha) - s_3 \sin(\alpha) \\
p_1 \\
-s_3 \cos(\alpha) + c_3 \sin(\alpha) \\
p_1 \\
p_1 \\
d_4 \\
p_6
\end{bmatrix}
\]

According to the low-level control design in Section 3, a control allocation scheme can be defined in order to optimally distribute the control effect among the thrusters. Specifically, one can consider the high-level control scheme

\[
\tau_{all} = \arg \min_{\tau} (\omega_1 \tau_1^2 + \omega_2 \tau_2^2 + \omega_3 \tau_3^2 + \omega_4 \tau_4^2)
\]

subject to \( \bar{g}(z_p) \Gamma \tau = u_{eq} + u_n \), where the weights \( \omega_i \) are nonnegative. The solution can be expressed by means of
the weighted pseudo inverse $\tau_{all} = ([\hat{g}(zp)G]^{T})^\dagger \Omega(u_{eq} + u_{n})$, where $[\hat{g}(zp)G]^{T}G\Omega^{T}\hat{g}(zp)G\Omega^{T}\hat{g}(zp)G]^{T}^{-1}$, $\Omega = \text{diag}(\omega_1, \omega_2, \omega_3, \omega_4)$.

4.1 Fault diagnosis

Class of faults: The faults are modeled as actuator loss of efficiency. In this regard, let us introduce the efficiency matrix

$$\Delta = \text{diag}(\delta_1, \delta_2, \delta_3, \delta_4), \quad \delta_j \in [0, 1] \quad j = 1, ..., 4,$$

such that the actual actuator state is $\tau_{eff} = \Delta \tau_{alloc}$. In particular in a fault-free scenario $\delta_1 = \delta_2 = \delta_3 = \delta_4 = 1$, or equivalently $\Delta = I$, and hence the identity $\tau_{eff} = \tau_{alloc}$ holds. On the other hand, in the case of a partial loss of efficiency, e.g. $\delta_j \in (0, 1)$, or total failure, e.g. $\delta_j = 0$, the allocation scheme ceases to produce the desired thrust effect. A wider class might include also faults affecting the actuator own dynamics, leading to potential faulty steady-states (Cristofaro and Johansen, 2014a).

Assumption 3. Only one of the four thrusters can undergo a fault, i.e. multiple thruster faults cannot be admitted. Moreover, it is assumed that any fault does not compromise controllability of the plant driven by the remaining healthy thrusters.

Assumption 4. In view of the fact that the reaching phase can be made arbitrarily short, it is assumed that faults can occur only after a sliding motion has been achieved on (6).

Fault Detection: The detection of a fault and the identification of the failed actuator can be performed by means of simple considerations exploiting the ROV model. To this purpose, it is important to notice that the control law is computed imposing the achievement of a sliding motion of the observed surface (6). Therefore, once the sliding motion is established (i.e. when $\tilde{s}_i = 0$ after the reaching phase), it is straightforward to verify that any deviation of the sliding surface is due to the occurrence of a fault. For instance, if a fault occurs at the time $t_0$ on the thruster $\tau_j$ such that the control input actually supplied undergoes a deviation $\Delta \tau_j(t - t_f)$ for $t > t_f$ with respect to its theoretical value, for the first sliding surface it holds:

$$\tilde{s}_1(t) = \tilde{s}_1(t_f)$$

having denoted by $t_f$ the time when the sliding motion is achieved (therefore $\tilde{s}_1(t) = 0$ for $t > t_f$).

Proposition 5. Consider the uncertain ROV model (3) driven by the robust controller (7). Suppose that the thruster $T_{k}, k \in \{1, ..., 4\}$ undergoes a fault at time $t_f$, thus causing a deviation of $\Delta \tau_k(t - t_f)$ of the control input supplied with respect to its theoretical value. Then one has, for $t > t_f$:

$$\tilde{s}_1(t) = \frac{1}{p_1} \int_{t_f}^{t} \Delta \tau_1(\sigma - t_f) \cos(z_3 - (1)^{k+3} \alpha) d\sigma$$

$$\tilde{s}_2(t) = \frac{1}{p_1} \int_{t_f}^{t} \Delta \tau_2(\sigma - t_f) \sin(z_3 - (1)^{k+3} \alpha) d\sigma$$

$$\tilde{s}_3(t) = \frac{d_2}{p_6} \int_{t_f}^{t} (1)^k \Delta \tau_4(\tau - t_f) d\sigma$$

where the symbol $\div$ denotes the operator of division between integers.

Proof. The statement follows directly from Assumption 4 and from the observers (5).

From the previous proposition, a sufficient fault detection rule immediately follows.

**Proposition 6.** Consider the uncertain ROV model (3) driven by the robust controller (7) under Assumptions 3,4. Suppose that the thruster $T_k, k \in \{1, ..., 4\}$ underwent a fault at time $t_f$. The fault can be detected checking the variables $\tilde{s}_i(t), i = 1, 2, 3$, at $t < t_f$, i.e. if

$$(\tilde{s}_1(t) \neq 0) OR (\tilde{s}_2(t) \neq 0) OR (\tilde{s}_3(t) \neq 0)$$

then a fault has occurred.

Proof. The proof is straightforward. It simply consists in checking the eventual violation of the sliding mode existence condition, according to (11). It is worth recalling that, according to Assumption 4, the sliding motion has been established, and the sliding surface (6) should be zero in the absence of a fault affecting the actuators.

Fault Isolation: The identification of the thruster which underwent the fault can be performed by means of simple considerations exploiting the ROV structure. Assume a fault has occurred at time $t_f$, and define for $t > t_f$:

$$I_{c1}(t) = \int_{t_f}^{t} \cos(z_3 + \alpha) d\sigma; \quad I_{c2}(t) = \int_{t_f}^{t} \sin(z_3 + \alpha) d\sigma; \quad I_{c3}(t) = \int_{t_f}^{t} (\cos(z_3 + \alpha) - \sin(z_3 + \alpha)) d\sigma.$$

Proposition 7. Consider the uncertain ROV model (3) driven by the robust controller (7), under Assumptions 3,4. Suppose that the thruster $T_k, k \in \{1, ..., 4\}$ underwent a fault at time $t_f$, then detected at time $t_d > t_f$. Compute the quantities for $t > t_d$:

$$\mu_{11}(t) = \frac{\tilde{s}_1(t)}{I_{c1}(t)}; \quad \mu_{13}(t) = \frac{\tilde{s}_1(t)}{I_{c3}(t)}; \quad \mu_{22}(t) = \frac{\tilde{s}_2(t)}{I_{c2}(t)}; \quad \mu_{24}(t) = \frac{\tilde{s}_2(t)}{I_{c4}(t)}; \quad \mu_{35}(t) = \frac{\tilde{s}_3(t)}{I_{c5}(t)}; \quad \mu_{36}(t) = \frac{\tilde{s}_3(t)}{I_{c6}(t)}.$$

(12)

The failed thruster $\tau_f$ can be isolated according to the following rule. Fix a time $t > t_d$.

- If $\text{sign} (\mu_{11}(t)) \neq \text{sign} (\mu_{13}(t))$ then $\mu_{12}(t) = \mu_{11}(t)$ and $\tau_f = \tau_1$ else $\tau_f = \tau_j$;
- else if $\text{sign} (\mu_{11}(t)) = \text{sign} (\mu_{13}(t))$ then $\mu_{22}(t) = \mu_{11}(t)$ and $\tau_f = \tau_2$ else $\tau_f = \tau_j$;

Proof. The statement follows directly from Proposition 5. In fact for a short interval $(t_f, t)$, assuming that the loss of effectiveness is occurring slowly enough, the term $\Delta \tau_k(\sigma - t_f)$ in (11) can be moved outside the integral signs. It follows that comparing signs of the quantities (12) one can identify the failed actuator, exploiting the model (10). Just as an example, suppose that the thruster $\tau_2$ underwent a fault of intensity $\Delta \tau_2(\tau - t_f)$. According to the model (10), one has

$$\tilde{s}_1(t) = \frac{1}{p_1} \Delta \tau_2 \int_{t_f}^{t} \cos(z_3 + \alpha) d\sigma$$

$$\tilde{s}_2(t) = \frac{1}{p_1} \Delta \tau_2 \int_{t_f}^{t} \sin(z_3 + \alpha) d\sigma$$

$$\tilde{s}_3(t) = \frac{d_2}{p_6} \Delta \tau_2 (t - t_f)$$

(13)
therefore $\mu_{11} = \Delta \tau_2$, $\mu_3 = \Delta \tau_2(t - t_f)$, and $\mu_{11}, \mu_3$ have the same sign. The same would anyway have occurred if the fault had undergone in the thruster $t_1$, in view of the structure of the matrix $g(z_3)$ of the model (10). To discriminate between $t_2$ and $t_3$, it is enough to consider, from the first two equalities of (13):
\[
\frac{1}{p_1} \Delta \tau_2 = \frac{\tilde{s}_1(t)}{\int_{t_f}^t \cos(z_3 - \alpha) d\sigma} = \frac{\tilde{s}_3(t)}{\int_{t_f}^t \sin(z_3 - \alpha) d\sigma}
\]
This approach can be generalized to the remaining possible cases. In particular, in the case when $\text{sign}(\mu_{11}(t)) \neq \text{sign}(\mu_3(t))$, only thrusters $t_1$ or $t_3$ could have experienced a fault. Moreover, if $\mu_{22}(t) = \mu_{11}(t)$, then the failed actuator is $t_1$, otherwise is $t_3$. An analogous argument holds for the case when $\text{sign}(\mu_{11}(t)) = \text{sign}(\mu_3(t))$, for which the candidate failed actuators are $t_2$ or $t_4$.

4.2 Adaptive control re-allocation

After a failure has been detected and isolated by the FD and FI module respectively, control reconfiguration is applied to preserve the desired performances in face of the failure occurrence. While in previous works this has been supposed as part of the human supervisor duty (see for instance (Corradini and Orlando, 2014)), in this paper we propose an automated reconfiguration method. In particular, the inherent redundancy of the considered ROV can be exploited for fault accommodation using an adaptive scheme for the allocation weight matrix $\Omega$. As output of the FD1 module one gets two indices, namely $j^*$ and $j^*$, such that
\[
|\tilde{s}_{j^*}(t)| = \max_{i=1,2,3} |\tilde{s}_i(t)| > 0,
\]
and $j^*$ is the faulty actuator.

The aim is to adjust the weights $\omega_1, \omega_2, \omega_3, \omega_4$ in order to progressively shift the control load on the safe actuators $t_j$, $j \neq j^*$. The signal $|\tilde{s}_{j^*}(t)|$ will play the role of an indicator of the fault severity and will be used accordingly to tune the adaptation learning rate. Let us formalize these ideas. Denote by $t_*$ the control reconfiguration initialization time and set $\Omega_0 = \text{diag}(\omega_{0,1}, \omega_{0,2}, \omega_{0,3}, \omega_{0,4})$ such that
\[
\Omega = \begin{cases} 
\Omega_0 & t < t_* \\
\Omega(t) & t \geq t_* 
\end{cases}
\]
(14)
where $\Omega(t) = \text{diag}(\tilde{\omega}_1(t), \tilde{\omega}_2(t), \tilde{\omega}_3(t), \tilde{\omega}_4(t))$ is defined as
\[
\dot{\tilde{\omega}}_j = -|\tilde{s}_{j^*}(t)| \tilde{\omega}_j, \quad \tilde{\omega}_j = 0, \quad j \neq j^*
\]
(15)
with $\Omega(t^*) = \Omega_0$. According to such adaptation law, the larger is the deviation from the sliding surface due to the fault in the actuator $t_{j^*}$, the larger is the rate of decay of the corresponding weight $\omega_{j^*}$. The following statement is therefore straightforward.

Proposition 8. Consider the ROV model (10), with the low-level control law (7) and the high-level re-configured allocation scheme (14)-(15). Then, two scenarios are admissible:

- **Actuator performance recovery:** there exists $t^* > t_*$ such that $|\tilde{s}_{j^*}(t)| = 0$ for $t \geq t^*$, and hence $|\tilde{\omega}_{j^*}(t)| = 0$ for $t \geq t^*$.

- **Actuator shutdown:** one has $|\tilde{s}_{j^*}(t)| > 0$ for any $t \geq t^*$, with the asymptotic conditions
\[
\lim_{t \to \infty} |\tilde{s}_{j^*}(t)| = 0, \quad \lim_{t \to \infty} |\tilde{\omega}_{j^*}(t)| = 0.
\]

Remark 9. The first case does correspond to the most likely scenario where, reducing the faulty actuator contribution in the overall thrust generation but keeping it still running, is sufficient to recover the desired tracking performances. Conversely, the actuator shutdown case can be regarded to as a worst-case scenario, where the fault is too severe to allow a partial redistribution of the thrust forces, and therefore a total ban of the faulty device is needed to accomplish the control task.

5. SIMULATION RESULTS

The proposed actuator fault tolerant control scheme has been validated by simulation. Tests have been performed in the following operative condition: 1) The measurements of position and orientation are subject to white noise; 2) Parameters variations of 10% with respect to their nominal value (Corradini and Orlando, 1997), (Corradini et al., 2011); 3) In the simulation tests, the plant initial condition has been chosen as $x(0) = 0$, $y(0) = 0$, $\phi(0) = 0$, $\dot{x}(0) = 0$, $\dot{y}(0) = 0$, $\phi(0) = 0$, and the set point as $y_d = (1 \text{ m} \text{ m}^2/\text{s})$. Favorable submarine current has been considered, with $V_c = [0.1, 0.1]^T$ m/s. Notice that such marine currents have been considered constant since they are very slowly time-varying due to the fact that they model submarine currents at great sea depth. 5) The sliding mode controller has been implemented including boundary layers with the aim of reducing chattering. Accordingly, the fault detection logic does correspond to deviation from boundary layer thresholds. 6) The actuator $t_2$ has been supposed to undergo a fault at $t = 70$ s, with a consequent loss of effectiveness. Before fault occurrence, the control action applied to the ROV is subdivided among the four thrusters (one of which is redundant). Results have been reported in Figs. 2-6. It can be verified that, before the fault occurrence on $t_2$, actuators $t_2, t_3$ and $t_4$ are able to effectively control the ROV (see Fig. 2-4). Moreover, simulations results show that satisfactory performances are maintained also in the faulty situation, since the ROV controlled outputs effectively follow the reference values (see Fig. 2) also after fault occurrence, and that observation and tracking errors are bounded (see Figs. 3-4). It is interesting to verify that detection of the fault is correctly performed by the sliding surfaces at $t = 70$ s (see Fig. 6), since the sliding surfaces noticeably deviate from the boundary layer thresholds. After fault isolation, control re-allocation is performed according to the adaptation law (15), with a progressive deactivation of the faulty thruster $t_2$ and activation of $t_1$ so that, along with $t_3$ and $t_4$, does ensure output regulation and maintain the required control performances. The changes in actuator states are depicted in Fig. 5 where, in particular, the gradual reduction of the $t_2$ control effect and the gradual fading in of the actuator $t_1$ are clearly visible.

REFERENCES


