A First Step Toward a Possibilistic Swarm Multi-Robot Task Allocation

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Abstract. The task allocation problem is one of the main issues in multi-robot systems. Typical ways to address this problem are based on Swarm Intelligence. One of them is the so-called Response Threshold Method. In the aforementioned method every robot has associated a task response threshold and a task stimuli in such a way that the robot's probability of executing a certain task depends on both factors. On of the advantage of the aforesaid method is given by the fact that the original problem is treated from a distributed mode which, at the same time, means a very low computational requirements. However, the Response Threshold Method cannot be extended in a natural way to allocate more than two tasks when the theoretical basis is provided by probability theory. Motivated by this fact, this paper leaves the probabilistic approach to the problem and takes a first step towards a possibilistic theoretical approach in order to treat successfully the multi-robot task allocation problem when more than two tasks must be performed. As an example of application, an scenario where each robot task stimuli only depends on the distance between tasks is studied and the convergence of the system to an stable state is shown.

Keywords: Multi-robot, Possibility Theory, Swarm Intelligence, Task allocation

1 Introduction

Systems with two or more mobile robots (multi-robot-systems) can perform tasks that with only one robot would be impossible to carry out or would take a lot of time. Moreover, such systems are more robust, scalable and flexible than those with only one robot. A lot of new challenges and problems must be solved before taking advantage of the potential benefits of the multi-robot systems. Among all possible issues that arise in a natural way in multi-robot systems, this paper focuses on the problem commonly referred to as "Multi-robot Task Allocation" problem (MRTA for short) which consists of selecting the best robot to execute each of the tasks that must be performed. A lot of research has been done to solve the aforementioned problem in the last years. Concretely, many efforts have been made to propose suitable methods based on auction and swarm strategies for task allocation.

Auction paradigms [9, 6] are based on explicit communication protocols between robots: when a robot, called auctioneer, finds or generates a new task sends a message to announce it before starting the execution. The other robots, called bidders, send to the auctioneer a value, called bid, that indicates how suitable is itself for executing the new task. Once the auctioneer has received all the bids, it selects the robot with the highest bid for the task. Auction methods, and negotiation paradigms in general, provide better solutions than swarm approaches. Nevertheless, the communication requirement in such methods can become a drawback.

Swarm methods are inspired by insect colonies behavior, where the cooperative behavior emerges from the interaction of very simple behaviors running on each robot without any communication protocol [15]. Thus, swarm methods are more scalable and simple than auction mechanisms. Because of these advantages a lot of swarm like algorithms has been posed but, nowadays, those based on the so-called Response Threshold Method (RTM for short) are probably the most broadly used (see Section 2.2 for a detailed description of the classical swarm algorithm based on RTM). In these methods, each involved robot has an associated a task response threshold and a task stimuli.The task stimuli value changes over the time and indicates how much *atarctive* is the task for the robot. When the task stimuli, associated to a robot, takes a value greater than a certain threshold, the robot starts the execution of the task or, as happens in most cases, a robot selects a tasks with a probability functions that depends on the stimuli itself and the threshold [2]. To our best knowledge, in this system there are only two tasks and the robots can only choose between them. Hence, a robot can stay on its current task or change to the other one [16]. As can be seen, this decision making process can be dealt as a Markov chain with only two states. However, the probabilistic approach which yields support to Markov chains can become a handicap. In fact, if the robots has a number of tasks greater than two, such as happens in a real mission, then either the RTM based algorithms cannot be directly applied or to fit the transition probabilities in the Markov model can be a very hard labor and, hence, it is necessary to reason by means imprecise probabilities that are fixed subjectively. In addition to this handicap, the applicability of the Markov approach to real missions is also reduced because of transition probabilities must meet some typical constraints. One of them is that of the transition probabilities from one state to the other ones must be a probability distribution (the sum of all of them must be equal to 1) and, in general, this constraint is not satisfied in real problems. Despite, it is worthy to point out that in the literature can be found Markov chain decision processes in which normalized transition probabilities have been proposed in order to model the behaviour of multi-robot systems where there are more than two possible tasks (see, for instance, [13]). Although, this methodology implies to introduce unnatural manipulations of the original system. Furthermore, in real situations the transitions can be represented by numerical values outside the range of the unit interval. So the probability theoretical foundation may even be inappropriate.

In the light of the above-mentioned inexpedients associated to the probabilistic RTM for task allocation, it seems natural to search through literature a theoretical formalism, with a basis different from the probability theory, that may be useful to study in a natural way the task allocation problem via a RTM when more than two tasks are under consideration and, in addition, does not involve artificial requeriments or constraints as, for instance, the normalization. Fortunately, the desired formalism can be found in the literature and it is known as possibility theory (see [3] and [17]). For this reason this paper takes a first step towards a possibilistic theoretical formalism for a RTM and its utility for the MRTA problem. Concretely, the RTM will be implemented considering transitions possibilities instead of transitions probabilities and this fact will imply that in the intrinsic decision process the possibilistic Markov chains (also known as fuzzy Markov chains) play the role of the probabilistic ones. Moreover, a few powerful properties of this new method will be showed.

The remainder of the paper is organized as follows: Section 2 reviews the basics of the MTRA problem. Thus the relevant notation and the problem statement are introduced in Subsection 2.1. In Subsection 2.2 the swarm approach to the MTRA problem is presented. Concretely, the classical swarm algorithm based on the RTM is detailed and one limitation is discussed. Section 3 is devoted to developed the possibilistic theoretical formalism for a RTM and to show its utility for the MRTA problem. Specifically, in Subsection 3.1 formalizes a few concepts about possibilistic theory and fuzzy Markov chains that will play a crucial role in order to achieve our objective. Moreover, in Subsection 3.2, a specific MTRA problem is approached from a Swarm viewpoint via the use of fuzzy Markov chains and some relevant properties of such a method are studied. Besides, some typical cases of study are discussed in order to illustrate the utility of the new method. Finally, the conclusions and future work are presented in Section 4.

2 Multi-Robot Task Allocation Problem

2.1 The problem

In the literature there are a lot of MRTA problem definitions and all of them depending on the characteristics of the problem to solve (see [7]). One criteria, among others, to classify the MRTA problems consider the number of robots that can be assigned to each task [7]. Thus, if two or more robots can collaborate to carry out the same task, the problem is called “Multi-Robot Task” problem (MRT for short). Otherwise, if only one robot can be assigned to each task at the same time, the problem is called “Single-Robot Task” problem (SRT for short). This paper only considers, as a first approach to the possibilistic MRTA problem, the SRT problems that can be defined as follows.

Let $\mathbb{N}$ denote the set of positive integer numbers and let $n, m \in \mathbb{N}$. Denote by R the set of robots with $R = \{r_1, ..., r_n\}$ and by T the set of tasks to carry out with $T = \{t_1, ..., t_m\}$. A task allocation is a function $TA : T \to R$ such that $T(t_i) \bigcap T(t_j) = \emptyset$ provided that $i \neq j$. Observe that T assigns to each task $t_j \in T$ a robot $r_i \in R$ in such a way that no more than one robot can be assigned to the same task. Thus, the goal of a task allocation algorithm is to find an optimal task allocation TA^*, among all valid TA functions, which optimizes some system characteristics.

Following [5], the SRT problems can be described in terms of the well-known Optimal Assignment Problem (OAP). The OAP is defined in the following way:

Consider n robots (or agents in general) and m tasks to carry out, each robot can only be assigned to one task and each task requires only one robot. For each couple (robot-task), a value is defined that forecasts the robots performance for that task, that is, this value models the robots utility regarding that task. The goal is to assign a robot to each task to maximize the total utility U. This goal function is given by

$$U = \sum_{1 \leq i \leq n} \sum_{1 \leq j \leq m} \alpha_{ij} U_{ij} w_j, \tag{1}$$

where $\alpha_{ij} = 1$ if the task i is assigned to agent j, and $\alpha_{ij} = 0$ otherwise; U_{ij} is the utility gained by the system when agent j is assigned to task i; and w_j is the weight or importance of the task j. Thus, w_j represents the priority of the task. The Hungarian's method [14] allows to get the optimal solution to this kind of problems in a time $O(nm^2)$ through a dynamic programing centralized method.

2.2 A Swarm Task Approach to MTA Problem: The Response Threshold Method

Although the Hungarian's method gives an optimal assignment which solves the OAP and, thus, the SRT problem, it is worthy to point out that it requires a central agent with global knowledge about the characteristics of all the robots and tasks to be performed. Moreover, since the environment where the robots are operating is dynamic (it can change over the time) the allocation algorithm should be executed constantly. Since in most of the real tasks it is needed a decentralized and very low computational cost task allocation algorithm, the preceding fact poses a handicap which is against the use of the Hungarian's method. In this direction methods based on swarm intelligence, as the RTM, are more useful and appropriate.

According to [1], the classical response threshold method defines for each robot r_i and for each task t_j, a stimuli $s_{r_i,t_j} \in \mathbb{R}$ that represents how suitable t_j is for r_i, where $\mathbb{R}$ stands for the set of real numbers. When s_{r_i,t_j} exceeds a given threshold ($\theta_{r_i} \in \mathbb{R}$), the robot r_i starts to execute the task t_j. To avoid relying on the threshold value to an excessive degree, the task selection is usually probabilistic. Thus, a robot r_i will select a task t_j to execute with a probability P_{r_i,t_j} which is given by

$$P_{r_i,t_j} = \frac{s^n_{r_i,t_j}}{s^n_{r_i,t_j} + \theta^n_{r_i}} \tag{2}$$

Figure 1 shows Equation (2) values as a function of s_{r_i,t_j} for several values of the exponent n and with $\theta_{r_i} = 50$. In order to reproduce the conditions usev by most of the authors, n will be always equal to 2 (see [11]).

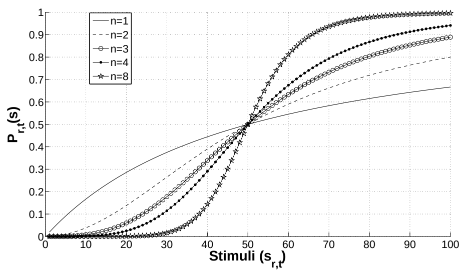

Fig. 1. P_{r_i,t_j} values (Equation 2) as a function of the stimuli for several n values with $\theta_{r_i} = 50$
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As mentioned earlier, the classical response threshold method is only suitable when the robots have, at a given time, two available actions or tasks to perform, i.e., $T = \{t_1, t_2\}$. Therefore the probability P_{r_k,t_i,t_j} that the robot r_k ($k = 1 \ldots, n$) leaves the task t_i in order to perform the task t_j, with $i, j \in \{1, 2\}$ and $i \neq j$, can be calculated according to Equation (2) by

$P_{r_k,t_i,t_j} = \frac{s^2_{r_k,t_j}}{s^n_{r_k,t_j}+\theta^2_{r_k}}$ and $P_{r_k,t_i,t_i} = 1 - P_{r_k,t_i,t_j}$.

Of course, the evolution of the system over the time can be modeled by means of a finite Markov chain where, for each robot r_k, the transition matrix $P_{r_k} = \{Pr_k, ij\}^n_{i,j=1}$ is given by $Pr_k, ij = P_{r_k,t_i,t_j}$ for all $i, j = 1, 2$. Notice that, for each $k, i \in \{1, 2\}$, $P_{r_k,i1} + P_{r_k,i2} = 1$.

It is clear that the preceding approach fits perfectly to the case of two possible states of the system, i.e., $T = \{t_1, t_2\}$. However, when more tasks are available to be performed by the each robot over the time, i.e. $T = \{t_1, \ldots, t_m\}$ ($m > 2$) then it seems natural to ask whether in general, for each robot r_k and $i \in \{1, \ldots, n\}$, the equality $\sum_{j=1}^m P_{r_k,t_i,t_j} = 1$ holds. Obviously this constraint is violated in many real situations (see, for instance, [8]). In order to avoid this disadvantage, as we have mentioned in Section 1, normalization processes can be introduced although they imply to impose unnatural system modifications. Moreover, in addition to the aforesaid inconveniences, to determine the transitions probabilities is an arduous task in general and, therefore, reasoning with imprecise probabilities, which have to be fitted subjectively, becomes needful to model the behavior of the system. Finally, let us recall that when more than two tasks are under consideration then the system will evolve to a stable state provided that, for each robot r_k, the transition matrix P^{r_k} is regular (see [10]). In this case the behavior of the systems is described by the following result.

Theorem 1. *Let P be a transition matrix for a regular Markov chain. Then the matrices P^n approach a limiting matrix W with all rows the same vector w whose all components are strictly positive and their sum is equals 1. Moreover, $wP = w$.*

In the light of the preceding result, it must be pointed out that the evolution of the system to a stable state is only guaranteed asymptotically and, as a consequence, in many cases the knowledge of the final state of the systems is obtained by successive approximations. It follows that in the probabilistic framework the final state of the systems is known with some degree of approximation in many cases.

3 Possibility Theory and Task Allocation

Since the transition probabilities in multi-task allocation problems can violate in a natural way a few axioms of the probability theory and they may be also imprecise and fixed subjectively, in this section a RTM based on a more general framework than the probabilistic one is introduced. Concretely a possibilistic approach for task allocation is proposed in such a way that multi-task allocation problems can be formulated via RTM based techniques in which the above-mentioned handicaps disappear. The aforementioned framework allows to encode the imprecise nature of the transitions of the system and to model the transitions probabilities without incorporating unnatural manipulations in the spirit of the normalization process.

3.1 Possibility Theory and Markov Chains

Next we recall a few pertinent notions from the possibility theory which will be crucial to achieve our announced target.

On account of [3] (see also [17]), given a non-empty finite set Ω, a possibility distribution on Ω is a function $Pos : \Omega \to [0,1]$ such that $\max_{\omega \in \Omega} Pos(\omega) \leq 1$. Moreover, provided that the power set of Ω is denoted by $\mathcal{P}(\Omega)$, a (non-normalized) fuzzy measure on Ω is a function $\mathcal{M} : \mathcal{P}(\Omega) \to [0,1]$ which holds the following axioms:

i) $\mathcal{M}(\emptyset) = 0$;
ii) $\mathcal{M}(\Omega) \leq 1$;
iii) $\mathcal{M}(A) \leq \mathcal{M}(B)$ provided that $A \subseteq B$.

A fuzzy measure $\mathcal{M}$ on Ω is called a possibility measure whenever the additional axiom is satisfied:

iv) $\mathcal{M}(A \cup B) = \max\{\mathcal{M}(A), \mathcal{M}(B)\}$ for all $A, B \in \mathcal{P}(\Omega)$.

Of course, a possibility distribution Pos on Ω induces a non-normalized fuzzy measure on Ω denoted by $\mathcal{M}_{Pos} : \mathcal{P}(\Omega) \to [0, 1]$ and defined by

$$\mathcal{M}_{Pos}(A) = \max_{\omega \in A} Pos(\{\omega\})$$

for all $A \in \mathcal{P}(\Omega)$. Notice that $\mathcal{M}_{Pos}(\Omega) = \max_{\omega \in \Omega} Pos(\omega) \leq 1$.

Usually the probability of an "event" $A \in \Omega$ is understood as a degree of likelihood or frequency which A occurs. In contrast, the possibility of A, $\mathcal{M}_{Pos}(A)$, is related to our perception of the degree of feasibility of A occurs. Thus, $\mathcal{M}_{Pos}(A) = \max_{\omega \in \Omega} Pos(\{\omega\})$ and $\mathcal{M}_{Pos}(A) = 0$ mean that the event A is totally possible or plausible and impossible, respectively. Nevertheless, contrarily to the probabilistic approach, there may be two events A and B with $\mathcal{M}_{Pos}(A) = \mathcal{M}_{Pos}(B) = \max_{\omega \in \Omega} Pos(\{\omega\})$, i.e., both events are totally possible. Despite these notable differences between possibility and probability, there is a relationship between them. In particular, if an event is probable then it is certainly possible or, equivalently, the degree of possibility of 3an event is greater than or equal to its degree of probability.

The theory of possibility has been applied to model those decision making processes based on Markov chains recently in [12]. Next we recall the basic notions about possibilistic Markov chains (fuzzy Markov chains in [12]) because they will be crucial in our subsequent discussion.

Consider a system a system evolving in time in such a way that the states of the system is fixed and finite, say $S = \{s_1, \ldots, s_k\}$. Moreover, at any unit of time the system changes from one state to another one according the following memoryless possibilistic law:

If the system is in the state s_i at time n ($n \in \mathbb{N}$), then the system will move to the state s_j with possibility p_{ij} at time $n+1$. Moreover, the transition possibility p_{ij} does not depend upon which states the system was in before the current state s_i. Of course, this law yields that the future state of the system depends only on the present state. Furthermore, given that the system is in the state s_i at time n then the system will move to one of the $s_1, \ldots, s_k$ possible states at time $n+1$ and, hence, we have that $\bigvee_{j=1}^{k} p_{ij} \leq 1$ for every $1 \leq i \leq n$, where $\vee$ stands for the maximum operator on $[0, 1]$.

Note that the numerical value $\bigvee_{j=1}^{k} p_{ij}$ provides information about what is the most possible state at which the system that the system will move from the state s_i. Although, of course, several states can become enjoy the same degree of possibility.

Next, let $x_i(n)$, $1 \leq i \leq k$, denote the possibility that the state s_i will occur at time n. Then it follows that $\bigvee_{i=1}^{k} x_i(n) \leq 1$ and, in addition, such a numerical value can be understood as the evolution of the system at time n is governed by the most possible state at that time. Notice that one of the states $s_1, \ldots, s_k$ must occur at time n. However, two states can become equally possible at time n and, thus, the dynamics of the system will be totally determined by the possibilities of transition. Obviously, the evolution of the system in time is given by the following equations

$$x_i(n) = \bigvee_{j=1}^{k} p_{ji} \wedge x_j(n-1) \tag{3}$$

for all $i = 1, \ldots, k$ and for all $n \in \mathbb{N}$, where $\wedge$ stands for the minimum operator on $[0, 1]$.

Obviously, the preceding systems of equations is equivalent to the below one in matrix notation:

$$x(n) = x(n-1) \circ P \tag{4}$$

for all $n \in \mathbb{N}$, where $P = \{p_{ij}\}_{i,j=1}^{n}$ is the matrix of transition possibilities, $\circ$ is the matrix product in the max-min algebra $([0, 1], \vee, \wedge)$ and $x(n) = (x_1(n), \ldots, x_k(n))$ for all $n \in \mathbb{N}$. Observe that $x(n) = (x_1(n), \ldots, x_k(n))$ represents the possibility distribution of the set of states at time n (the ith component of $x(n)$ matches up with the possibility of the system is in state s_i at time n. Naturally, a possibility distribution $x(n)$ of the system states at time n is said to be stationary, or stable, whenever $x(n) = x(n) \circ P$.

The great advantage of the possibilistic Markov chains with respect to the probabilistic ones is given by the fact that under certain conditions the system which is modeled will converge to a stable (stationary) state in a finite time. In contrast probabilistic Markov chains under the hypothesis of regularity guaranteed that the system will converge to a stationary state asymptotically, i.e., not necessarily in a finite time (see Theorem 1). The conditions that provide the finite convergence character of the system in the possibilistic case can be found in [4]. With the aim of recalling such conditions let us introduce a few notions about fuzzy matrix.

Following [4], a matrix $A \in M_n([0, 1])$ will said to be k-power-convergent if $A^k = A^{k+1}$ for some $k \in \mathbb{N}$, where A^k denotes the max-min composition of A and itself k times. Moreover, the least $k \in \mathbb{N}$ such that A is k-power-convergent (or simply power-convergent) will be denoted by $k(A)$ and called the index of A. Furthermore, given $A, B \in M_n([0, 1])$, we denote by $A \leq B$ the fact that $a_{ij} \leq b_{ij}$, where $\leq$ stands for the usual order on $[0, 1]$. Finally, a matrix $A \in M_n([0, 1])$ will said to be column diagonally dominant provided that $a_{ii} \geq a_{ji}$ for all $i, j = 1, \ldots, n$.

Taking into account the above introduced concepts we are able to state the result which guarantees the aforesaid finite convergence character of possibilistic Marckov chains.

Theorem 2. *Let $A \in M_n([0, 1])$. Assume that A is column diagonally dominant and that $A \leq A^2$. Then A is power-convergent and $k(A) \leq n - 1$.*

In the light of the preceding result, those systems modeled by possibilistic Markov chains whose transition matrix satisfies the assumptions in the stament of Theorem 2 will evolve to a stationary state in a finite time.

It must be pointed out that the preceding result will be crucial in our subsequent discussion, since the equation that describes the evolution of a possibilistic Markov chain, Equation (4), is equivalent to the following one: $x(n) = x(0) \circ P^n$ for all $n \in \mathbb{N}$. So the study of the power-convergence of the transition matrix will play a central role in the study of the behavior of the system under consideration.

3.2 Possibilistic Markov chains and Task Allocation

This section shows an example of how possibilistic Markov chains are useful for developing a RTM for solving multi-robot task allocation problems. For the sake of simplicity, and honoring the the titile of the paper, a very simple but representative case is discussed. Finally, two illustrative examples, which worth to mention, are studied.

According to the MRTA problem statement consider a collection of robots $R = \{r_1, ..., r_n\}$ $(n \in \mathbb{N})$ and a set of tasks to carry out $T = \{t_1, ..., t_m\}$ $(m \in \mathbb{N})$. Assume that the tasks are randomly placed in an environment and the robots are initially randomly placed. Then the target is to find an optimal task allocation in such a way that only one robot can be assigned to each task at the same time. Hence our possibilistic RTM must decide which task must execute each robot although this decision must be made individually by each robot without exchanging information between them. Moreover, we will assume that each robot allocation only depends on the distance between the robot and the task.

From now on, denote by $x_i(n) = (x_{i1}(n), ..., x_{im}(n))$ a fuzzy set, where $x_{ij}(n)$ is the possibility for the robot r_i of executing the task t_j at time n. Consider the position space endowed with a metric d. Then denote by $d(r_i, t_j)$ the distance between the current position of r_i and the position of t_j and by $d(t_i, t_j)$ the distance between the position of the task t_i and t_j. Of course, it is assumed that when a robot is assigned to a task the position of this task and the robot's position are the same and therefore, the distance between the task and the robot is 0. Following the response-threshold notation, define the stimuli of the robot r_k to carry out task t_j as follows:

$$s_{r_k,t_j} = \begin{cases} \frac{1}{d(r_k,t_j)} & \text{if } d(r_k,t_j) \neq 0 \\ \frac{1}{\alpha} & \text{if } d(r_k,t_j) = 0, \end{cases} \tag{5}$$

where $\alpha = min_{i,j=1,...,m} d(t_i, t_j)$. This stimuli s_{r_k,t_j} allows us to obtain, by means of Equation (2), the response threshold possibility

$$p_{r_k,ij} = \begin{cases} \frac{1}{1+d(r_k,t_j)^2\theta^2_{r_k}} & \text{if } d(r_k,t_i) = 0 \text{ and } d(t_i,t_j) \neq 0 \\ \frac{1}{1+\alpha^2\theta^2_{r_k}} & \text{if } d(r_k,t_i) = 0 \text{ and } d(t_i,t_j) = 0. \end{cases} \tag{6}$$

Notice that the numerical value $p_{r_k,ij} = \frac{1}{1+\alpha^2\theta^2_{r_k}}$ in Equation (6) can be understood in the following way: a robot r_k tends to stay on the task t_i when its position coincides with that of the task t_i. So the perception of the degree of feasibility of the robot stays at the position of the task t_i at the next time should be related to the greatest numerical value of the assigned possibilities. However, it is also possible that the robot r_k leaves the position of the task t_i, despite its current position coincides with that of the task t_i, and moves to the position of the task t_j to carry out it. This perception of the degree of feasibility that event occurs is represented by the numerical value $p_{r_k,ij} = \frac{1}{1+d(r_k,t_j)^2\theta^2_{r_k}}$. Note that when smaller the distance $d(t_i, t_j)$, with $i \neq j$ and $d(r_k, t_i) = 0$, the greater is the possibility degree of feasibility of the robot r_k transits from the position of t_i to the position of t_k.

From Equation (6) one can compute the fuzzy matrix of possibilities $P_{r_k} = \{p_{r_k,ij}\}^m_{i,j=1}$ which enjoys nice properties such as the next result shows.

Proposition 1. *Fix $r_k \in R$. Then P_{r_k} is power-convergent with $k(P_{r_k}) \leq m-1$.*

Proof. The fact that $0 \leq p_{r_k,ij} \leq 1$ for all $i, j = 1, \ldots, m$ immediately yields that $P_{r_k} \in M_m([0, 1])$. Moreover, P_{r_k} is column diagonally dominant and satisfies that $P_{r_k} \leq P^2_{r_k}$, since $P^2_{r_k ij} = \bigvee^n_{k=1}(p_{il} \wedge p_{lj})$ for all $l = 1...m$. Therefore, by Theorem 2, we conclude that P_{r_k} is power-convergent with $k(P_{r_k}) \leq m - 1$.

As a consequence of the preceding result we obtain the following conclusion.

Corollary 1. *Fix $r_k \in R$. Then the possibilistic Markov chain with transition matrix P_{r_k} converge to a stationary non-periodic solution in at most $m-1$ steps and, in addition, such a convergence does not depend on the initial posibilistic distribution $x_i(0)$.*

Finally, we focus our attention on two illustrative and interesting cases of study.

Case Study 1: Homogeneous robots and one robot per task.

In this context homogeneous robots means that all the robots have the same threshold, that is, $\theta_{r_i} = \theta$ for $i = 1...n$. Initially each robot r_k is assigned to a different task with a possibility $\frac{1}{\alpha}$. If we assume the same number of robots and tasks, $n = m$, then the robot r_k can initially be assigned to task t_k for all $k = 1, \ldots, n$. Moreover, it is clear that $p_{r_k,ij} = p_{r_l,ij}$ for all $k, l = 1, \ldots, n$. Furthemore, the initial possibility distribution of the states for robot r_k is given by

$$x_k(0) = (x_{k1}(0), \ldots, x_{kn}(0)),$$

where

$$x_{ki}(0) = \begin{cases} \frac{1}{\alpha_k} & \text{if } k = i \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

By Corollary 1 we have that the possibility distribution of sates for each robot will converge to a stationary non-periodic one in at most $n-1$ steps. In particular if case we take $n=3$, then the possibility distribution of states for both robots will converge to a stationary non-periodic one in at most 2 steps. Specifically the possibility distribution of states at step 2 for each robot will be the following one:

$$\begin{array}{l} x_1(2) = (\frac{1}{\alpha}, \bigvee(p_{12}, p_{13} \wedge p_{32}), \bigvee(p_{13}, p_{12} \wedge p_{23})) \\ x_2(2) = (\bigvee(p_{21}, p_{23} \wedge p_{31}), \frac{1}{\alpha}, \bigvee(p_{23}, p_{21} \wedge p_{13})) \\ x_3(2) = (\bigvee(p_{31}, p_{32} \wedge p_{21}), \bigvee(p_{32}, p_{31} \wedge p_{12}), \frac{1}{\alpha})) \end{array}$$

Although all robots have the same transition matrix, all of them converge to a different possibilistic distribution of states. Despite in all cases the most possible state is that of the robot stays performing the task where it was initially allocated, the robots have a non zero possibility for other tasks. Thus, all task has a possibility greater than 0 of execution.

Case of Study 2. Heterogeneous robots and all the robots are assigned to the same task.

In this case all the robots has the same initial possibility distribution of states, for example all robots are initially allocated to task t_1. But, now the robots are heterogeneous, that is, $\theta_{r_i} \neq \theta_{r_j}$ provided $i \neq j$. Therefore each robot will have its own transition matrix P_{r_k}. Note that the number of robots and task may differ but if we take $n=m$, then, by Corollary 1, we have that the possibility distribution of sates for each robot will converge to a stationary non-periodic one in at most $n-1$ steps.

In particular if we take $n=m=3$, then the possibility distribution of states for all robots will converge to a stationary non-periodic one in at most 2 steps. Specifically if we assume, as an example, that all robots are allocated to task t_1, then the possibility distribution of states at step 2 for each robot will be the following one:

$$x_k(2) = (\frac{1}{\alpha}, \bigvee(p_{r_k,12}, p_{r_k,13} \wedge p_{r_k,32}), \bigvee(p_{r_k,13}, p_{r_k,12} \wedge p_{r_k,23}))$$

for all $k = 1, \ldots, n$.

If the threshold had been the same for all the robots, all of them would converge to the same state, generating a non balanced system, that is, with a high possibility all the robots would have executed the same task (t_1). But now the robots are heterogeneous, so each vector $x_k(2)$ depends on θ_{r_k} and, therefore, the final distribution of the robots can be fitted through the threshold values.

4 Conclusions and Future Work

This paper has taken a first step to develop the theoretical basis for possibility multi-robot task allocation methods based on swarm intelligence. One of the most important swarm MRTA method is the response threshold. As mentioned in this paper, RT methods have a few inconveniences, from practical and theoretical point of view. In fact, if there are more than 2 states then easily either to determine the transitions probabilities is a hard task and, therefore, they have to be fitted subjectively, or the distribution of transitions is not probabilistic and must be normalized. This handicaps can be avoided using possibilistic Markov chains which clearly offers a more realistic and general approach because allow to model imprecise probabilities. The paper proves that, in the specific case where the transition possibility depends on a distance, the Markov process converges in a finite number of steps lower or equal to the tasks.

This paper presents a work in its very first stages that has a lot of challenging aspects to add and to improve. For the time being, we focus on new scenarios and possibility transitions that depends on new factors like for example the utility of the task. We also plan to compare possibilistic Markov chains with current response threshold methods from an empirical point of view.

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