A Comparative Analysis of Indistinguishability Operators Applied to Swarm Multi-Robot Task Allocation Problem

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Abstract One of the main problems to solve in multi-robot systems is to select the best robot to execute each task (task allocation). Several ways to address this problem have been proposed in the literature. This paper focuses on one of them, the so-called response threshold methods. In a recent previous work, it was proved that the possibilistic Markov chains outperform the classical probabilistic approaches when they are used to implement response threshold methods. This previous study only takes into account a celebrated possibility response/transition function. In this paper we use a new possibility transition function and we make several experiments in order to compare both, the new one and the tested before. The experiments show that the number of steps that a possibilistic Markov chain needs to converge does not depend on the response function used. In contrast, the aforementioned number seems to be very related to the placement of the tasks in the environment and it differs when each aforesaid response functions are under consideration. This paper also emphasizes that theses possibility transition functions are indistinguishably operators and, thus, that indistinguishability operators could be useful in the modeling of response functions in Swarm Multi-Robot Task Allocation Problem.

Key words: Indistinguishability Operator, Markov Chain, Multi-robot, Possibility Theory, Swarm Intelligence, Task allocation

1 Introduction

Multi-robot systems, and in general multi-agent systems, are defined as systems with two or more robots (or agents) that perform the same mission or task. These systems provide several advantages regarding single-robot systems, for example: robustness, flexibility and efficiency. To make its benefits several problems have to be addressed. This paper focuses on the problem commonly referred to as “Multi-robot Task Allocation” (MRTA for short) which consists in selecting the best robots to execute each one of the tasks that must be performed.

MRTA is still an open problem and due to its significance, a lot of research has been done to solve this problem in the last years (see [10]). Some of the proposed solutions are based on swarm intelligence, where the cooperative behaviour emerges

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from the interaction of very simple behaviours running on each robot. Due to its simplicity, scalability and robustness, several swarm like algorithms have been proposed but, nowadays, those based on the so-called Response Threshold Method (RTM for short) are probably the most broadly used. In these methods, each involved robot has associated a task response threshold and a task stimuli. The task stimuli value indicates how much attractive is the task for the robot. When the task stimuli, associated to a task, takes a value greater than a certain threshold, the robot starts its execution following a probability function. This is a Markov process, where the probability of executing a task only depends on the current task (state). This probabilistic approach presents a lot of disadvantages: problems with the selection of the probability function when more than two tasks are considered, asymptotic converge, and so on.

In the light of the above-mentioned inexpedient associated to the probabilistic RTM for task allocation, in [9] a new possibilistic theoretical formalism for a RTM was proposed and its utility for the MRTA problem was also proved. In this case, the RTM is implemented considering transitions possibilities instead of transitions probabilities and this fact implies that in the intrinsic decision process, the possibilistic Markov chains (also known as fuzzy Markov chains), play the role of the probabilistic ones. The theoretical and empirical results demonstrated that fuzzy Markov chains applied to task allocation problem require a very few number of steps to converge to a stable state. In all cases the transition possibility from one task to another one was modeled by a widely accepted response function (see (1)). This paper extends our previous work about RTM and studies the impact of other kind of transition possibilistic functions, concretely the exponential one. An extensive number of experiments has been carried out with different threshold values and several kind of tasks distributions using Matlab. These experiments show that the convergence time does not depend on the possibility transition function (response function) used. However, it depends on the distribution of the tasks in the environment and, thus, it differs when each aforesaid response functions are under consideration. It must be stressed that theses possibility transition functions are both indistinguishably operators in the sense of [13] and, thus, that indistinguishability operators could be useful in the modelling of response functions in Swarm Multi-Robot Task Allocation Problem.

The remainder of the paper is organized as follows: Section 2 reviews the basics of the MTRA problem. Section 3 shows the experiments carried out to validate our approach and, finally, Section 4 presents the conclusions and future work.

2 Multi-Robot task allocation

This section introduces the main concepts about multi-robot task allocation and the RTM approaches and review the previous works made in this field.

2.1 Probabilistic response threshold methods

As pointed out in Section 1, the response threshold methods are a very promising approach in order to face up realistic tasks in a decentralized way. According to [1], the classical response threshold method defines for each robot r_i and for each task t_j, a stimuli $s_{r_i,t_j} \in \mathbb{R}$ that represents how suitable t_j is for r_i, where $\mathbb{R}$ stands for the set of real numbers. When s_{r_i,t_j} exceeds a given threshold θ_{r_i} ($\theta_{r_i} \in \mathbb{R}$), the robot r_i starts to execute the task t_j. To avoid relying on the threshold value to an excessive degree, the task selection is usually modeled by a probabilistic response function. Thus, a robot r_i will select a task t_j to execute with a probability $P(r_i,t_j)$ according to a probabilistic Markov decision chain. There are different kind of probabilities response functions that defines a transition, but one of the most widely used (see [4, 15]) is given by

$$P(r_i,t_j) = \frac{s^n_{r_i,t_j}}{s^n_{r_i,t_j} + \theta^n_{r_i}}, \tag{1}$$

where $n \in \mathbb{N}$, where $\mathbb{N}$ stands for the set of natural numbers. It must be pointed out that the preceding response function has been also used in [9]. In this paper we test another transition function that presents similar characteristics to the given in (1) and which is given by:

$$P(r_i,t_j) = \mathrm{e}^{-\frac{\theta^n_{r_i}}{s^n_{r_i,t_j}}} \tag{2}$$

It is not hard to check that the above-mentioned transitions functions are indistinguishably operators whenever s_{r_i,t_j} only depends on the distance between the robot and the task as follows:

$$s_{r_i,t_j} = \frac{1}{d(r_i,t_j)}.$$

We refer the reader to [13] for the definition of this kind of operators. Moreover, in general for both response functions, the equality $\sum_{j=1}^m P(r_k,t_j) = 1$ does not hold and, hence, the transition does not meet the axioms of the probability theory. In order to avoid this disadvantage normalization processes can be introduced although they imply to impose system modifications with possible implications in the behavior of the system.

2.2 Possibilistic Markov chains: theory

As was proved in [9], possibilistic Markov chains provide a lot of advantages and outperform its probabilistic counterpart. This section summarizes the main theoretical concepts of possibilistic Markov chains and the new aforementioned (possibility) exponential transition response function is introduced.

Following [2, 16] we can define a possibility Markov (memoryless) process as follows: let $S = \{s_1,\ldots,s_m\}$ ($m \in \mathbb{N}$) denote a finite set of states. If the system is in the state s_i at time τ ($\tau \in \mathbb{N}$), then the system will move to the state s_j with possibility

p_{ij} at time $\tau+1$. Let $x(\tau) = (x_1(\tau),...,x_m(\tau))$ be a fuzzy state set, where $x_i(\tau)$ is defined as the possibility that the state s_i will occur at time τ for all $i = 1,\ldots,m$. Notice that $\bigvee_{i=1}^{m} x_i(\tau) \leq 1$ where $\vee$ stands for the maximum operator on $[0,1]$. In the light of the preceding facts, the evolution of the fuzzy Markov chain in time is given by

$$x_i(\tau) = \bigvee_{j=1}^{m} p_{ji} \wedge x_j(\tau-1),$$

where $\wedge$ stands for the minimum operator on $[0,1]$. The preceding expression admits a matrix formulated as follows:

$$x(\tau) = x(\tau-1) \circ P = x(0) \circ P^{\tau}, \tag{3}$$

where $P = \{p_{ij}\}_{i,j=1}^{m}$ is the fuzzy transition matrix, $\circ$ is the matrix product in the max-min algebra $([0,1],\vee,\wedge)$ and $x(\tau) = (x_1(\tau),\ldots,x_m(\tau))$ for all $\tau \in \mathbb{N}$ is the possibility distribution at time τ.

Taking into account the preceding matrix notation and following [2], a possibility distribution $x(\tau)$ of the system states at time n is said to be stationary, or stable, whenever $x(\tau) = x(\tau) \circ P$.

One of the main advantages of the possibilistic Markov chains with respect to their probabilistic counterpart is given by the fact that under certain conditions, provided in [5], the system converges to a stationary state in a finite number of steps.

2.3 Possibilistic Response Threshold

In this section we will see an example of how to use possibilistic Markov chains for developing a RTM in order to allocate a set of robots to tasks. We will assume that the tasks are randomly placed in an environment and the robots are initially randomly placed too. Furthermore, we will assume that each robot allocation, that is the stimulus, only depends on the distance between the robot and the task. Consider the position space endowed with a distance (metric) d. Then, denote by $d(r_i,t_j)$ the distance between the current position of r_i. It is assumed that when a robot is assigned to a task the position of this task and the robot's position are the same and therefore, the distance between the task and the robot is 0. Following the RTM notation, define the stimulus of the robot r_k to carry out task t_j as follows:

$$s_{r_k,t_j} = \begin{cases} \frac{U_{t_j}}{d(r_k,t_j)} & \text{if } d(r_k,t_j) \neq 0 \\ \infty & \text{if } d(r_k,t_j) = 0 \end{cases}. \tag{4}$$

This stimulus s_{r_k,t_j} allows us to obtain, by means of (1), the following possibility response function,

$$p_{r_k,ij} = \frac{U_{t_j}^{n}}{U_{t_j}^{n} + d(r_k,t_j)^{n}\theta_{r_k}^{n}}. \tag{5}$$

If the same stimulus s_{r_k,t_j} is used in (2), then the following exponential possibility response function is obtained:

$$p_{r_k,ij} = e^{-\frac{\theta_{r_k}^n d(r_k,t_j)^n}{U_{t_j}^n}}. \quad (6)$$

For convenience to our subsequent discussion we will reference the response function given by (6) as Exponential Possibility Response Function (EPRF for short) and the response function given by (5) as Original Possibility Response Function (OPRF for short). As stated in Section 2.1, both possibility response functions are also indistinguishably operators. Therefore, we will use interchangeably the concepts of indistinguishably operator and possibility response/transition functions when we reference the former.

In [9], it was demonstrated that the response function given by (5) fulfills smooth conditions (column diagonally dominant and power dominant) that guarantee the finite convergence (see [5] for a detailed description of such notions) provided that all tasks have the same utility U_{t_j}. Therefore, fixed $r_k \in R$, the possibilistic Markov chain obtained by means of the OPRF converges to a stationary non-periodic state in finite time (exactly in at most $m-1$ steps). Following similar arguments to those given in [9] one can prove easily that the possibilistic Markov chain obtained by means of EPRF also converges to a stable state in at most $m-1$ steps.

3 Experimental Results

In this section we will explain the experiments performed to compare the number of steps required to converge to a stationary state using probabilistic and possibilistic Markov chains induced from the indistinguishability operators given in (5) and (6).

3.1 Experimental Framework

The experiments have been carried out under different conditions: position of the objects (placement of tasks), parameters of the possibility response functions (θ_{r_k} and n) and number of tasks. All the experiments have been carried out using MATLAB with different synthetic environments following a uniform distribution to generate the position of the tasks. Figure 1 represents a set of experiments where the task have been placed randomly in the environment, where each blue dot is a task. Furthermore, all the environments have the same dimension (width=600 units and high=600 units). In the shake of simplicity, we assume that all the tasks have the same utility, i.e, $U_{t_j} = 1$ for all $j = 1, \ldots, m$.

Following the reasoning made in [11], the θ_{r_k} must depend on the environment conditions. During the performed experiments the θ_{r_k} will depend on the maximum distance between tasks as follows:

$$\theta_{r_k} = \frac{nTH}{d_{max}}, \quad (7)$$

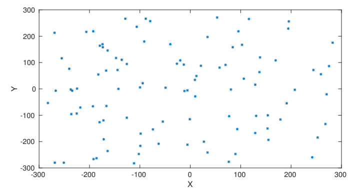


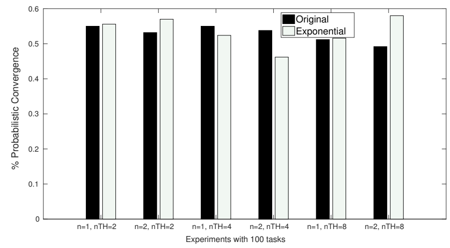
Fig. 1 Kind of environments with 100 task randomly placed. Blue dots represents the position of the tasks or objects.

where d_{max} is the maximum distance between two objects and nTH is a parameter of the system. Due to the above-mentioned environment dimensions, d_{max} is constant and equals to 800.5 units. For the first time, the nTH parameter has been introduced with respect to previous papers (see [11]) in order to analyze how the threshold value impact on the system performance. All the experiments have been performed with 500 different environments, with different number of tasks ($m = 50, 100$) and different values of the power n in the expression of possibility response functions (see (5) and (6)). The threshold θ_{r_k} values under consideration are obtained from (7) setting $nTH = 2, 4, 8$.

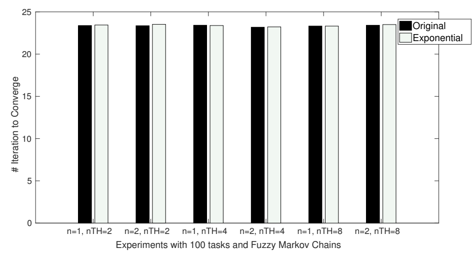
Whichever possibility response function is used, the given by either (5) or (6), the possibilistic transition matrix, P_{r_k}, must be converted into a probabilistic matrix, in order to be comparable the possibilistic and probabilistic Markov chain results. To make this conversion we use the transformation proposed in [14], where each element of P_{r_k} is normalized (divided by the sum of all the elements in its row) meeting the conditions of a probability distribution.

Figure 2 shows some results obtained with 100 randomly placed tasks using OPRF given by (5), and EPRF given by (6). Figure 2(a) shows the percentage of experiments that with the probabilistic Markov process does converge. If the process does not converge after 10,000 iterations we assume that it will never converge. As can be seen, in all probabilistic cases around 50% of experiments converges. Although it is not graphically represented, the experiments that converges need 256.8 steps on average to do it. Figure 2(b) shows the mean number of steps required to converge using fuzzy Markov chains. Let us recall that, all fuzzy Markov chains under consideration converge. As can be seen, there are no significance differences between the experiments that use OPRF (Original labeled bars) and those that use EPRF (Exponential labeled bars). Moreover, the nTH parameter or the power value n do not have any impact on this results. Therefore, we can conclude that when the tasks are randomly placed in the environment, both possibility response functions provide similar results on average and both seems not to be affected by its parameters nTH or n. Moreover, the possibilitics Markov chains, whichever possibility response functions is used, needs much lower number of steps to converge compared to their probabilsitic counterpart.

Table 1 shows the standard deviation, σ, of the number of steps required to converge with 100 tasks when fuzzy Markov chains are considered. Thus, this table represents the standard deviation of the results given in Figure 2(b). From these



(a) Percentage of experiments that, using probabilistic Markov process, do converge.



(b) Number of steps required to converge with fuzzy Markov chains

Fig. 2 Experimental results with 100 tasks.

results, we can state that in most cases the use of EPRF produces a decrease of the standard deviation compared to the values of σ obtained when ORPF is used. Similar results, regarding mean and standard deviation, have been obtained after performing experiments with 50 tasks ($m = 50$).

Function	n=1, nTH=2	n=2, nTH=2	n=1, nTH=4	n=2, nTH=4	n=1, nTH=8	n=2, nTH=8
OPRF	3.59	3.48	3.56	3.58	3.7	3.52
EPRF	3.58	3.6	3.35	3.57	3.47	3.47

Table 1 Standard deviation (σ) of the number of steps required to converge with fuzzy Markov chains.

4 Conclusions and Future Work

This paper has presented an empirical comparative analysis of two indistinguishability operators (or possibility response function) applied to the convergence of possibilistic Markov chains where the goal of the system is to allocate tasks to a colony of robots using response-threshold methodologies. As was proved in [9], possibilistic Markov chains outperforms its probabilistic counterpart when they are used to model response-threshold multi-robot systems. This paper extends the aforementioned work. In addition, it shows how the use of the aforementioned possibility response functions in modeling fuzzy Markov chains provides similar results and they are very robust with respect to their parameters. In the light of the obtained results a lot of new challenges, problems and improvements must be addressed as future work. For the time begin, we focus on provide a deeper analysis about how the position of the tasks impacts on the convergence time, both from theoretical and empirical point of view. Other conversions from possibilistic distribution to probabilistic are also under consideration. Furthermore, the implementation of these methods using real robots is carrying out by the authors.

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